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STUDY OF THE DYNAMIC CONTACT INTERACTION OF DEFORMABLE BODIES

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A new algorithm for solving dynamic contact problems involving deformable bodies is proposed. The algorithm is based on formulation of the boundary conditions for the contact interaction with allowance for Coulomb friction in the form of quasivariational inequalities. The algorithm is numerically stable and satisfies geometric constraints in the a priori unknown contact region and conditions specifying that the normal pressure be nonnegative and that the vectors describing tangential velocity and shear stress during slip be oppositely directed. Results are presented from calculations performed for a contact problem for an elastoplastic body in a two-dimensional formulation.

Dynamic contact problems of the theory of elasticity and plasticity with a contact region that is unknown beforehand and changes during deformation [1–5] have a broad range of application in connection with studies of the impact and piercing of barriers, explosive and hydro-explosive forging, machining, and other processes. Methods that are explicit with respect to time are usually used to calculate the contact boundaries in the numerical solution of such problems. The use of these methods inevitably results in intersection of the bodies undergoing deformation or violation of certain dynamic conditions in the contact region.

In this investigation, we devise an approach based on exact formulation of the boundary conditions of the contact with allowance for friction in the form of quasivariational inequalities [6, 7]. The use of this formulation makes it possible to construct universal and efficient iteration methods that are numerically stable and satisfy geometric constraints in the contact region and also conditions specifying that the normal pressure be nonnegative and the vectors of tangential velocity and shear stress during slip be oppositely directed.

1. Contact Boundary Conditions. First we shall examine the contact interaction of a deformable body with a perfectly rigid die occupying the spatial region $\varphi(\mathbf{x}) \leqslant 0$, which has a piecewise-smooth surface $\varphi(\mathbf{x}) = 0$. We assume that when the body is in the initial undeformed state, we can distinguish a part of its boundary S_c whose material points at each subsequent moment of time are either in contact with the die or are free of stresses. On the remaining part of the boundary, sufficiently general boundary conditions are satisfied but contact with the die is impossible. The displacement vector $\mathbf{u} = \mathbf{u}(t, \mathbf{x})$ in a Lagrangian description satisfies the geometric constraint $\varphi(\mathbf{x} + \mathbf{u}) \geqslant 0$ on S_c. Allowing for the expansion $\mathbf{u} = \mathbf{u}\big|_{t-\Delta t} + \mathbf{v}\Delta t + O(\Delta t^2)$ and performing certain transformations, we use this constraint to obtain an approximate constraint on the velocity vector of a point $\mathbf{v}\nabla\varphi(\mathbf{x} + \mathbf{u}\big|_{t-\Delta t}) \geqslant -\varphi(\mathbf{x} + \mathbf{u}\big|_{t-\Delta t})\,/\,\Delta t$. This second constraint can be used to numerically realize the contact conditions. By passing to the limit of Δt at $\Delta t \to 0$, we obtain the exact constraint

$$\mathbf{v}\nabla\varphi(\mathbf{x} + \mathbf{u}) \geqslant -\delta(\mathbf{x} + \mathbf{u}), \qquad \delta(\mathbf{x}) = \begin{cases} 0, & \varphi(\mathbf{x}) = 0, \\ +\infty, & \varphi(\mathbf{x}) > 0. \end{cases} \tag{1.1}$$

Following [6–8], we write the contact conditions with allowance for Coulomb friction in the form of an inequality

$$(\mathbf{v}^* - \mathbf{v})\boldsymbol{\sigma}_n + f|\sigma_{nn}|(|\mathbf{v}^*_\tau| - |\mathbf{v}_\tau|) \geqslant 0. \tag{1.2}$$

Here $\boldsymbol{\sigma}_n = \boldsymbol{\sigma}_n(t, \mathbf{x})$ is the stress vector on an area of the deformed surface of the body with the normal $\mathbf{n}$. The subscript τ denotes projections of the vectors onto the tangent plane, f is the coefficient of sliding

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friction, and $\mathbf{v}^* = \mathbf{v}^*(t, \mathbf{x})$ is an arbitrary permissible variation of velocity at a point of the boundary S_c that satisfies constraint (1.1). We note that the normal to the surface of the body in the contact region $S_t = \{\mathbf{x} \in S_c\colon \varphi(\mathbf{x}+\mathbf{u}) = 0\}$ is calculated from the formula $\mathbf{n} = -\nabla\varphi(\mathbf{x}+\mathbf{u}) \,/\, |\nabla\varphi(\mathbf{x}+\mathbf{u})|$. Thus, (1.1) actually contains only the normal component of the velocity vector v_n.

In accordance with the standard terminology, inequality (1.2) is quasivariational because the constraint depends on an unknown vector $\mathbf{u}$. This inequality expresses the principle that the virtual work done by a stress in the contact region, equal to the difference between the work of the surface stresses $\mathbf{v}^*\boldsymbol{\sigma}_n$ and the work of friction $-f|\sigma_{nn}|\,|\mathbf{v}_\tau^*|$, assumes the minimum value on the real velocity vector.

Let γ be the Lagrangian multiplier corresponding to constraint (1.1). In accordance with the Kuhn–Tucker theorem, in the presence of a constraint the above principle regarding the minimum of a function that is convex in relation to the vector $\mathbf{v}^*$ is equivalent to the principle of the unconditional minimum of the Lagrangian

$$L(\mathbf{v}^*, \gamma) = \mathbf{v}^*\boldsymbol{\sigma}_n + f|\sigma_{nn}|\,|\mathbf{v}_\tau^*| - \gamma \mathbf{v}^*\nabla\varphi(\mathbf{x}+\mathbf{u}).$$

Here the multiplier γ is nonnegative and equal to zero if $\varphi(\mathbf{x}+\mathbf{u}) > 0$.

In the case $\mathbf{v}_\tau \neq 0$, the condition of the minimum L in differential form leads to the equations $\sigma_{nn} = -\gamma|\nabla\varphi(\mathbf{x}+\mathbf{u})|$ and $\boldsymbol{\sigma}_{n\tau} = -f|\sigma_{nn}|\,\mathbf{v}_\tau \,/\, |\mathbf{v}_\tau|$. The derivative of the Lagrangian does not exist when $\mathbf{v}_\tau = 0$. In this case, it follows directly from the condition $L(\bar{\mathbf{v}}, \gamma) \geqslant L(\mathbf{v}, \gamma)$ ($\bar{\mathbf{v}}$ is an arbitrary vector whose normal component is equal to v_n) that $\bar{\mathbf{v}}_\tau\boldsymbol{\sigma}_{n\tau} + f|\sigma_{nn}|\,|\bar{\mathbf{v}}_\tau| \geqslant 0$. From here, with allowance for the inequality $|\bar{\mathbf{v}}_\tau\boldsymbol{\sigma}_{n\tau}| \leqslant |\bar{\mathbf{v}}_\tau|\,|\boldsymbol{\sigma}_{n\tau}|$, we obtain $|\boldsymbol{\sigma}_{n\tau}| \leqslant f|\sigma_{nn}|$. Thus, the quasivariational inequality (1.2) conforms exactly to the Coulomb friction law.

Since the approximate constraint contains the projection of the velocity vector in the direction $\hat{\mathbf{n}} = -\nabla\varphi(\mathbf{x}+\mathbf{u}|_{t-\Delta t}) \,/\, |\nabla\varphi(\mathbf{x}+\mathbf{u}|_{t-\Delta t})|$, in the quasivariational inequality intended for numerical realization of the contact conditions we need to replace the stress σ_{nn} by $\sigma_{n\hat{n}}$ and replace the velocity $\mathbf{v}_\tau$ by the projection $\mathbf{v}_{\hat{\tau}}$ of the velocity vector onto the plane with the normal $\hat{\mathbf{n}}$.

Of course, the exact formulation of the contact conditions is independent of the function φ that parameterizes the surface of the dies. A constraint equivalent to formula (1.1) can also be obtained by another method that does not require φ to be specified. In that method, we construct a special mapping $\boldsymbol{\pi}$ of the space onto the boundary or part of the boundary of the die for which each material point $\mathbf{x}+\mathbf{u}$ in the contact region is stationary and thus maps the area onto itself; an example would be a map of a design that associates each point of the space with the closest point of the die. If such a map is known, then the approximate constraint is written in the form

$$\mathbf{v}^*\hat{\mathbf{n}} \leqslant \left|\mathbf{x}+\mathbf{u}\Big|_{t-\Delta t} - \boldsymbol{\pi}\Big(\mathbf{x}+\mathbf{u}\Big|_{t-\Delta t}\Big)\right| \Big/ \Delta t, \tag{1.3}$$

where $\hat{\mathbf{n}}$ is a unit vector that is equal to the outer normal to the deformed surface of the body if the point $\mathbf{x}$ belongs to the contact region $S_{t-\Delta t}$ and is equal to the vector $-\big\{\mathbf{x}+\mathbf{u}|_{t-\Delta t} - \boldsymbol{\pi}\big(\mathbf{x}+\mathbf{u}|_{t-\Delta t}\big)\big\} \big/ \big|\mathbf{x}+\mathbf{u}|_{t-\Delta t} - \boldsymbol{\pi}\big(\mathbf{x}+\mathbf{u}|_{t-\Delta t}\big)\big|$ outside the contact region.

The above-described method is convenient for numerically solving dynamic contact problems if the computation is performed in time steps. In its practical realization, $\boldsymbol{\pi}(\mathbf{x})$ can be a mapping that makes the point $\mathbf{x}$ correspond to the distance from this point to the point of intersection of the boundary of the die either with a straight line which passes through $\mathbf{x}$ and through a certain fixed internal point of the die or with a straight line which is orthogonal to a prescribed smooth surface inside the die (Fig. 1).

The constraint (1.3) has a clear geometric interpretation. The exact constraint (1.1) can be obtained from it by passing to the limit of Δt.

We similarly formulate the contact conditions for two deformable bodies. Let S_c^+ and S_c^- be parts of the boundaries of these bodies in Lagrangian variables that completely envelop the contact regions $S_t^\pm = \{\mathbf{x}^\pm \in S_c^\pm\colon \mathbf{x}^+ + \mathbf{u}^+(t, \mathbf{x}^+) = \mathbf{x}^- + \mathbf{u}^-(t, \mathbf{x}^-)\}$ at each fixed moment of time t. We also assign the mapping $\boldsymbol{\pi}^-$ of a certain region of space onto the image of the boundary S_c^- in the deformed state. The mapping is dependent on time and the parameter of the problem and transfers each material point from the contact region back onto itself. Since the domain of the mapping $\boldsymbol{\pi}^-$ contains the deformed boundary S_c^+, we write the approximate geometric constraint for the velocity of the points $\mathbf{x}^+$ and $\mathbf{x}^- = \boldsymbol{\pi}^-(\mathbf{x}^+ + \mathbf{u}^+)\Big|_{t-\Delta t} - \mathbf{u}^-\Big|_{t-\Delta t}$

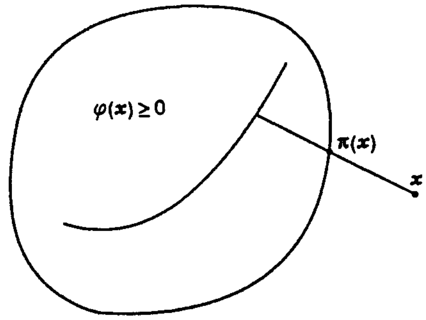

Fig. 1

in the form

$$(\mathbf{v}^{*+} - \mathbf{v}^{*-})\hat{\mathbf{n}} \leqslant |\mathbf{x}^+ + \mathbf{u}^+\big|_{t-\Delta t} - \mathbf{x}^- - \mathbf{u}^-\big|_{t-\Delta t}| \,/\, \Delta t, \tag{1.4}$$

where the vector $\hat{\mathbf{n}}$ is equal to $\mathbf{n}^+ = -\mathbf{n}^-$ for $\mathbf{x}^\pm \in S^\pm_{t-\Delta t}$ and is equal to the vector

$$-\left(\mathbf{x}^+ + \mathbf{u}^+\big|_{t-\Delta t} - \mathbf{x}^- - \mathbf{u}^-\big|_{t-\Delta t}\right) \Big/ \left|\mathbf{x}^+ + \mathbf{u}^+\big|_{t-\Delta t} - \mathbf{x}^- - \mathbf{u}^-\big|_{t-\Delta t}\right|$$

for points not belonging to $S^\pm_{t-\Delta t}$.

In the numerical realization, the mapping π^- for the moment of time $t - \Delta t$ can be constructed by one of the methods described above. The exact constraint on velocity is obtained from (1.4) with $\Delta t \to 0$:

$$\mathbf{v}^{*+}\mathbf{n}^+ + \mathbf{v}^{*-}\mathbf{n}^- \leqslant \begin{cases} 0, & \mathbf{x}^\pm \in S^\pm_t, \\ +\infty, & \mathbf{x}^\pm \notin S^\pm_t. \end{cases} \tag{1.5}$$

The contact conditions are represented in the form of a quasivariational inequality

$$(\mathbf{v}^{*+} - \mathbf{v}^+)\sigma^+_n + (\mathbf{v}^{*-} - \mathbf{v}^-)\sigma^-_n + (1/2)f(|\sigma^+_{nn} + \sigma^-_{nn}|)(|\mathbf{v}^{*+}_\tau - \mathbf{v}^{*-}_\tau| - |\mathbf{v}^+_\tau - \mathbf{v}^-_\tau|) \geqslant 0, \tag{1.6}$$

which can be interpreted as the principle of the minimum of the work done by a normal stress in the contact region. The equivalence of this principle and the Coulomb friction law for two deformable bodies can be established by means of the Kuhn–Tucker theorem, similarly to the case of the contact of a deformable body and a die. The approximate formulation of the conditions of contact interaction of the bodies corresponds to a quasivariational inequality obtained by replacing the normal stresses by the stresses $\pm\sigma^\pm_n\,\hat{\mathbf{n}}$ and replacing the difference between the tangential components of the velocity vectors by the projection of the vector $\mathbf{v}^+ - \mathbf{v}^-$ onto a plane orthogonal to $\hat{\mathbf{n}}$.

2. Algorithms for Correction of the Velocities. To numerically realize the contact conditions, we reduce the quasivariational inequalities (1.2) and (1.6) and the approximate constraints (1.3) and (1.4) to the following standard form:

$$(\mathbf{w}^* - \mathbf{w})A(\mathbf{w} - \bar{\mathbf{w}}) + f\,|\mathbf{b}(\mathbf{w} - \bar{\mathbf{w}})|\{\omega(\mathbf{w}^*) - \omega(\mathbf{w})\} \geqslant 0, \quad \mathbf{w}^*\mathbf{g} \geqslant h, \quad \mathbf{w}\,\mathbf{g} \geqslant h. \tag{2.1}$$

Here $\mathbf{w}$ is the m-dimensional vector to be determined. In the case of contact of a deformable body and a die, this vector contains the components of the velocity vector of a specified point, namely, the node of the grid region in cartesian coordinates. In the case of two deformable bodies, the m-dimensional vector contains the components of the velocity vectors of two nodes $\mathbf{x}^+$ and $\mathbf{x}^-$.

It is important to use linear equations in boundary nodes in making the transition from (1.2) and (1.6) to (2.1). This makes it possible to use known parameters at the moment of time $t - \Delta t$ to express the stresses at the moment t in terms of velocity at this moment. Such equations are needed when difference schemes that are explicit with respect to time are used to realize boundary conditions of any kind. The equations are usually constructed by approximating the relations for the bicharacteristics of the system of equations that describes the dynamic deformation of each of the interacting bodies. The coefficients of these equations determine the vector $\mathbf{b}$ and the square $m \times m$ matrix A, which is assumed to be symmetric and positive-definite. The assigned vector $\bar{\mathbf{w}}$ contains the components of the velocities corresponding to the formulation of the boundary conditions for the free surface at the nodes $\mathbf{x}^+$ and $\mathbf{x}^-$. The vector $\mathbf{g}$ and the scalar h

are obtained automatically from (1.3) and (1.4). The dimensionality m of the vectors that are introduced is determined by the dimensionality of the contact problem being examined (nondimensional, two-dimensional, and three-dimensional).

In the variational inequality (2.1), $\omega(\mathbf{w})$ represents the modulus of the tangent vector or the modulus of the difference between the projections of the velocity vectors of corresponding points on the tangent plane as a function of the type of interacting bodies. In the general case, the following representation is valid for this function:

$$\omega(\mathbf{w}) = \max_{l^* \in B} \mathbf{w}\,\mathbf{l}^* = \mathbf{w}\,\mathbf{l}, \tag{2.2}$$

where B is a bounded convex closed set of vectors of the dimensionality m. In the description of the contact of a deformable body and a die, the set B consists of vectors $\hat{\tau}$ that are orthogonal to the vector $\hat{\mathbf{n}}$ and whose length is no greater than unity. In the case of two bodies, the set B consists of vectors of the form $(\hat{\tau}, -\hat{\tau})$.

If by assigning the vector $\mathbf{w} = \hat{\mathbf{w}}$ in inequality (2.1) we fix the expression $c = f\,|\mathbf{b}(\mathbf{w} - \bar{\mathbf{w}})| \geqslant 0$, we arrive at the problem of minimizing the convex function $\psi(\mathbf{w}) = (1/2)(\mathbf{w}-\bar{\mathbf{w}})A(\mathbf{w}-\bar{\mathbf{w}}) + c\omega(\mathbf{w})$ on the convex closed set $K = \{\mathbf{w}\colon \mathbf{w}\,\mathbf{g} \geqslant h\}$. It can be shown that the mapping Q which associates the point representing the minimum of this function with the vector $\hat{\mathbf{w}}$ is compact for small values of the friction coefficients f.

In fact, let $\mathbf{w}' = Q(\hat{\mathbf{w}}')$. Then, if we assume that $\mathbf{w}^* = \mathbf{w}'$ in the variational inequality (2.1) for $\mathbf{w} = Q(\hat{\mathbf{w}})$ and that $\mathbf{w}^* = \mathbf{w}$ in a similar inequality for $\mathbf{w}'$ and if we add the results, we obtain

$$(\mathbf{w}' - \mathbf{w})A(\mathbf{w}' - \mathbf{w}) \leqslant f\,(|\mathbf{b}(\hat{\mathbf{w}} - \bar{\mathbf{w}})| - |\mathbf{b}(\hat{\mathbf{w}}' - \bar{\mathbf{w}})|)\{\omega(\mathbf{w}') - \omega(\mathbf{w})\}.$$

Performing the necessary transformations and taking into account the fact that the matrix A is positive definite, we then find that

$$\alpha|\mathbf{w}' - \mathbf{w}|^2 \leqslant f\,|\mathbf{b}|\,|\hat{\mathbf{w}}' - \hat{\mathbf{w}}|\,|\mathbf{w}' - \mathbf{w}|, \qquad \alpha = \min_{|w|=1} \mathbf{w}A\mathbf{w}.$$

Thus, the mapping Q is compact for $f < \alpha/|\mathbf{b}|$.

By virtue of the principle of compact mappings, in the given case there is a unique stationary point $\mathbf{w} = Q(\mathbf{w})$ that represents the sought solution of the variational inequality (2.1). This point can be found by a successive approximation. In each step of the approximation, we solve the problem of conditional minimization of the function $\psi(\mathbf{w})$ with a certain constant c.

In accordance with (2.2), the function being minimized is equal to

$$\psi(\mathbf{w}) = \max_{l^* \in B} \Psi(\mathbf{w}, \mathbf{l}^*), \qquad \Psi(\mathbf{w}, \mathbf{l}) = (1/2)(\mathbf{w} - \bar{\mathbf{w}})A(\mathbf{w} - \bar{\mathbf{w}}) + c\mathbf{w}\mathbf{l},$$

Here the function Ψ and the sets K and B are subject to the conditions of the saddle point theorem [9]. Thus, the conditional minimization problem can be formulated as a minimax problem:

$$\Psi(\mathbf{w}, \mathbf{l}) = \min_{w^* \in K} \max_{l^* \in B} \Psi(\mathbf{w}^*, \mathbf{l}^*) = \max_{l^* \in B} \min_{w^* \in K} \Psi(\mathbf{w}^*, \mathbf{l}^*).$$

We find the saddle point by using the algorithm developed by Udzava [9, 10]. We solve the problem of minimizing $\Psi(\mathbf{w}, \mathbf{l}^n)$ at the nth step of the algorithm with a fixed value of the dual vector $\mathbf{l} = \mathbf{l}^n$. Its solution $\mathbf{w} = \mathbf{w}^n$, satisfying the variational inequality

$$(\mathbf{w}^* - \mathbf{w}^n)\{A(\mathbf{w}^n - \bar{\mathbf{w}}) + c\,\mathbf{l}^n\} \geqslant 0, \qquad \mathbf{w}^n, \mathbf{w}^* \in K, \tag{2.3}$$

is found in explicit form after application of the Kuhn–Tucker theorem. Then using an operator that projects the solution orthogonally onto the set K with the norm $|\mathbf{w}|_A = \sqrt{\mathbf{w}A\mathbf{w}}$, we write the solution as $\mathbf{w}^n = P_K(\bar{\mathbf{w}} - cA^{-1}\mathbf{l}^n)$. After this, the vector $\mathbf{l}$ is recalculated in accordance with the formula $\mathbf{l}^{n+1} = P_B(\mathbf{l}^n + \tau\mathbf{w}^n)$, where P_B is an operator that projects onto the set B with an euclidean norm. The quantity $\tau > 0$ is an iteration parameter. The initial approximation $\mathbf{l}^0$ is assigned arbitrarily, and the computation is ended upon satisfaction of the condition $|\mathbf{l}^{n+1} - \mathbf{l}^n| \leqslant \tau\varepsilon$ (ε is the accuracy of the calculations).

We shall prove that as $n \to \infty$ the sequence of pairs of vectors $(\mathbf{w}^n, \mathbf{l}^n)$ converges to the saddle point $(\mathbf{w}, \mathbf{l})$ if $\tau < 2\alpha/c$. Choosing as the permissible variation $\mathbf{w}^*$ the vector $\mathbf{w}$ in inequality (2.3) and the vector $\mathbf{w}^n$ in a similar inequality for $\mathbf{w}$, after addition we obtain

$$\alpha|\mathbf{w}^n - \mathbf{w}|^2 \leqslant |\mathbf{w}^n - \mathbf{w}|_A^2 \leqslant -c(\mathbf{w}^n - \mathbf{w})(\mathbf{l}^n - \mathbf{l}).$$

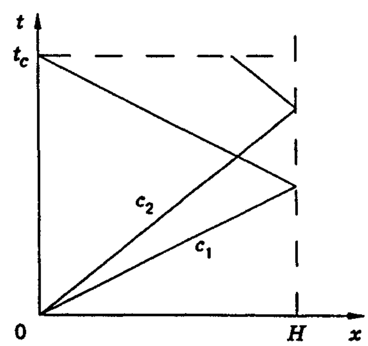

Fig. 2

On the other hand, since any projection operator for a convex set is a nonstretching transformation, we have $|\mathbf{l}^{n+1} - \mathbf{l}| \leqslant |\mathbf{l}^n - \mathbf{l} + \tau(\mathbf{w}^n - \mathbf{w})|$. Thus,

$$|\mathbf{l}^{n+1} - \mathbf{l}|^2 \leqslant |\mathbf{l}^n - \mathbf{l}|^2 + 2\tau(\mathbf{w}^n - \mathbf{w})(\mathbf{l}^n - \mathbf{l}) + \tau^2|\mathbf{w}^n - \mathbf{w}|^2 \leqslant |\mathbf{l}^n - \mathbf{l}|^2 - \tau(2\alpha/c - \tau)|\mathbf{w}^n - \mathbf{w}|^2.$$

It is apparent that the sequence of nonnegative numbers $|\mathbf{l}^n - \mathbf{l}|$ is decreasing and as $n \to \infty$ converges to a certain limit. It follows from this that $|\mathbf{w}^n - \mathbf{w}| \to 0$.

In the combined algorithm described above for numerically solving the variational inequality (2.1), the recursive calculation of the iterations is performed on the basis of the compact mapping Q and two nonstretching operators P_K and P_B. Such an algorithm is stable against round-off errors: the transition to the next iteration cannot lead to an increase in the errors. The computation can be limited to the so-called diagonal sequence, so that only k iterations of the Udzava algorithm have to be calculated at the kth step of the successive approximation.

3. Calculation Results. The algorithm was tested on a nondimensional problem concerning the interaction of an elastic layer of thickness H with a perfectly rigid surface. The vector of the initial velocity of the layer $\mathbf{v}^0$ was arbitrarily chosen. The exact solution of the problem is constructed by the method of characteristics with allowance for friction (Fig. 2). Longitudinal and transverse loading waves begin to propagate with the velocities c_1 and c_2 at the moment of contact inside the layer. The interaction of the longitudinal unloading wave and the contact surface at the moment of time $t_c = 2H/c_1$ leads to rebound of the layer. Here the velocity averaged over the thickness of the layer in the normal direction is equal to $-v_n^0$. The transverse wave affects the tangential velocity component after reflection. Slip occurs in the contact region for $c_2 v_\tau^0 > f c_1 v_n^0$. The tangential component of the mean velocity at the moment t_c is calculated as $v_\tau^0 - 2f v_n^0$. In the opposite case, the stagnation regime occurs: a state of rest is established behind the front of the transverse wave. In this regime, the tangential component of the mean velocity at the moment of reflection is found from the formula $v_\tau^0(1 - 2c_2/c_1)$. The mean velocity of the layer is directed perpendicularly to the surface for $c_2 = c_1/2$ and reverses direction for $c_2 > c_1/2$.

Calculations were performed for different regimes of interaction of the layer and the surface in accordance with the Godunov scheme for solving nondimensional problems of the dynamic theory of elasticity. Quantitative agreement was obtained within the limits of accuracy of the scheme.

We also examined a problem that models the symmetrical machining of a specimen by two perfectly rigid tools of cylindrical form. The problem was studied in a two-dimensional approximation. The calculations involved the use of a model that describes the dynamic deformation of elastic-ideally-plastic bodies with small strains. The numerical realization of the model was based on the two-dimensional Godunov scheme, with the use of a procedure to correct the stresses. In light of the symmetry of the problem, calculations were performed only for half of the specimen: it was assumed that the lower boundary of the region of the solution was the region of contact with the tool and that the upper boundary was the plane of symmetry. The load was applied to the right boundary, while the remaining parts of the contour were free of stresses.

Figure 3 shows results of calculations of the deformation of the specimen at certain moments of time for $f = 0$ (without allowance for friction) (a) and $f = 0.18$ (b). The plastic zone is hatched. Comparison of the

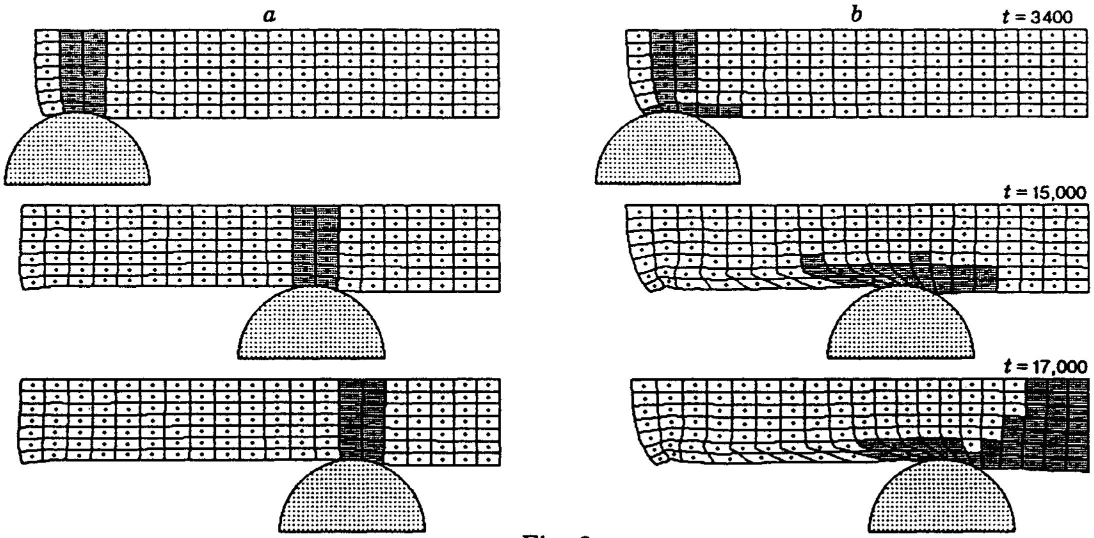

Fig. 3

results shows that friction significantly alters the structure of the solution. With large values for the friction coefficients, the plastic zone is stretched out along the contact surface. This stretching can be attributed to the presence of substantial shear stresses in the contact region. When the values of the friction coefficients are small, the contact region takes the shape of a vertical band associated with compressive transverse stresses caused by the action of the cylinders.

The calculations showed that the proposed algorithm is a reliable method of numerically solving problems involving the dynamic contact interaction of deformable bodies.

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